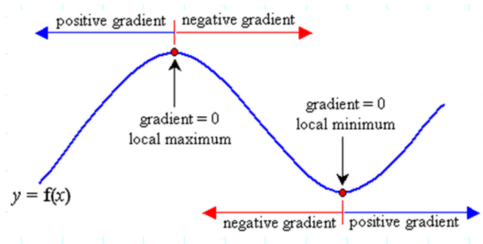
FEATURES OF GRAPHS

A) Turning points

- There are 2 kinds of turning points on a graph: maximum and minimum.
- To find turning points, solve f'(x) = 0, since gradient at turning points are zero.



B) Increasing and decreasing functions

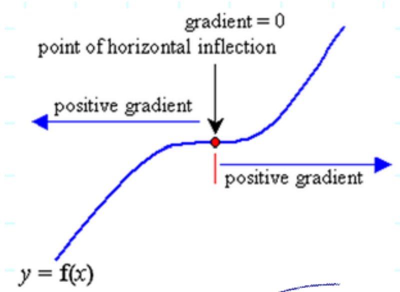
- A function is increasing if it has a positive gradient. To find the range of values for which the function is increasing, solve f'(x) > 0.
- A function is decreasing if it has a negative gradient. To find the range of values for which the function is decreasing, solve f'(x) < 0.

C) Stationary points

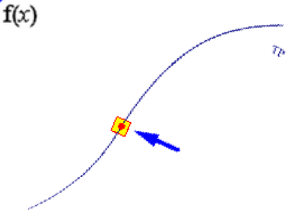
- A stationary point has a gradient of zero.
- By that definition, the two types of turning points (maximum and minimum) are stationary points.
- There is a third type of stationary point called a **point of inflection (p.o.i.)**.

POINTS OF INFLECTION

A point of inflection is a point where the graph changes in concavity, but on either side of the inflection point, the sign of the gradient stays the same (either positive on both sides of inflection point, or negative on both sides of inflection point).



Points of inflection are considered stationary points if their gradient is zero. Points of inflection can also have non-zero gradients (i.e. the curve does not flatten out when changing concavity)



CONCAVITY

All curves have concavity. A curve is concave down around a local maximum. 

A curve is concave up around a local minimum. 

SECOND DERIVATIVE TEST

The second derivative test (where you differentiate your function twice) is used to determine the nature of **stationary** points.

- If the second derivative is negative, the stationary point is a **maximum** point.
  f''(x) < 0 $\Rightarrow$ **maximum** point
- If the second derivative is positive, the stationary point is a **minimum** point.
  f''(x) > 0 $\Rightarrow$ **minimum** point
- If the second derivative is zero, the stationary point is a **point of inflection**.
  f''(x) = 0 $\Rightarrow$ point of inflection

To find points of inflection, which may or may not be stationary points, we solve the equation f''(x) = 0.

Question: For the curve $y = x^3 - 9x^2 + 24x - 19$,

a) Find the range of values for which the curve is increasing
b) Find the range of value for which the curve is decreasing
c) Calculate the coordinates of the stationary points and determine their nature.
d) Calculate the coordinates of the point(s) of inflection

a) f'(x) = 3x^2 – 18x + 24 = 3(x^2 – 6x + 8) = 3(x – 4)(x – 2)
   f(x) is increasing where f'(x) > 0 i.e. x < 2, x > 4.
b) f(x) is decreasing where f'(x) < 0 i.e. 2<x<4
c) Solve f'(x) = 0.
   3(x – 4)(x – 2) = 0
   x = 4, and x = 2 are stationary points.
   f''(x) = 6x – 18
   f''(4) = 6(4) – 18 = 6 > 0, so x = 4 is a minimum point (calculate the y-coordinate).
   f''(2) = 6(2) – 18 = -6 < 0, so x = 2 is a maximum point (calculate the y-coordinate).
d) Solve f''(x) = 0.
   6x – 18 = 0
   6x = 18
   x = 3 is a point of inflection.

Delta Ex 10.5 pg 112 Q1 – 2
Delta Ex 10.6 pg 112 Q1, Q3
Delta Ex 10.4 pg 102 Q1 - 2